Minimizing Sparse High-Order Energies by Submodular Vertex-Cover

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**Goal:** Minimize binary energies with unary potentials and very sparse high-order “pattern potentials.”

\[
F(x) = \sum_{i=1}^{9} a_i x_i - b_1 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 - b_2 x_2 x_3 x_4 x_5 x_6 x_7 x_8 + b_3 x_3 x_4 x_5 x_6 x_7 x_8 \]

1. **Submodular Vertex-Cover:** Generalizes vertex-cover to allow submodular costs [Iwata & Nagano, 2009].

\[
\begin{align*}
\text{minimize } f(u) \\
\text{subject to } u_i + u_j &\geq 1 & \forall (i,j) \in E \\
u_j &\in \{0,1\}.
\end{align*}
\]

**Typical Approach:** Convert to pairwise problem with auxiliary variables [e.g. Rother et al., 2009].

**Our Approach:** Transform to small SVC instance

1. **Transform to SVC**
2. **Inference (BP, QPBO-I, ...)**
3. **Inference (BP, QPBO-I, ...)**
4. **Decode covering**

**Synthetic Tests:** 100x100 grid, random unary potentials
50 random pattern potentials of strength \(\lambda\).

**Application:** Fast fusion operation for hierarchical model-fitting / clustering, e.g. scene parsing [Tretyak et al., 2011].

**Solution quality:** Range [best lower bound, ICM energy] normalised to [0,1] for each \(\lambda\).