Minimizing Sparse High-Order Energies by Submodular Vertex-Cover

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Goal: Minimize binary energies with unary potentials and very sparse high-order "pattern potentials."

e.g.
$$F(\boldsymbol{x}) = \sum_{i=1}^{9} a_i x_i - b_1 \bar{x}_1 \bar{x}_2 \bar{x}_3 \bar{x}_4 \bar{x}_5 - b_2 x_2 \bar{x}_3 \bar{x}_4 x_5 \bar{x}_6 - b_3 x_3 x_4 x_5 \bar{x}_6 \bar{x}_7 x_8$$

$x_i \in \{0,1\}$
$a_i \in \mathbb{R}$
$b_i \in \mathbb{R}_{\geq 0}$

individual preferences

rewards specific patterns

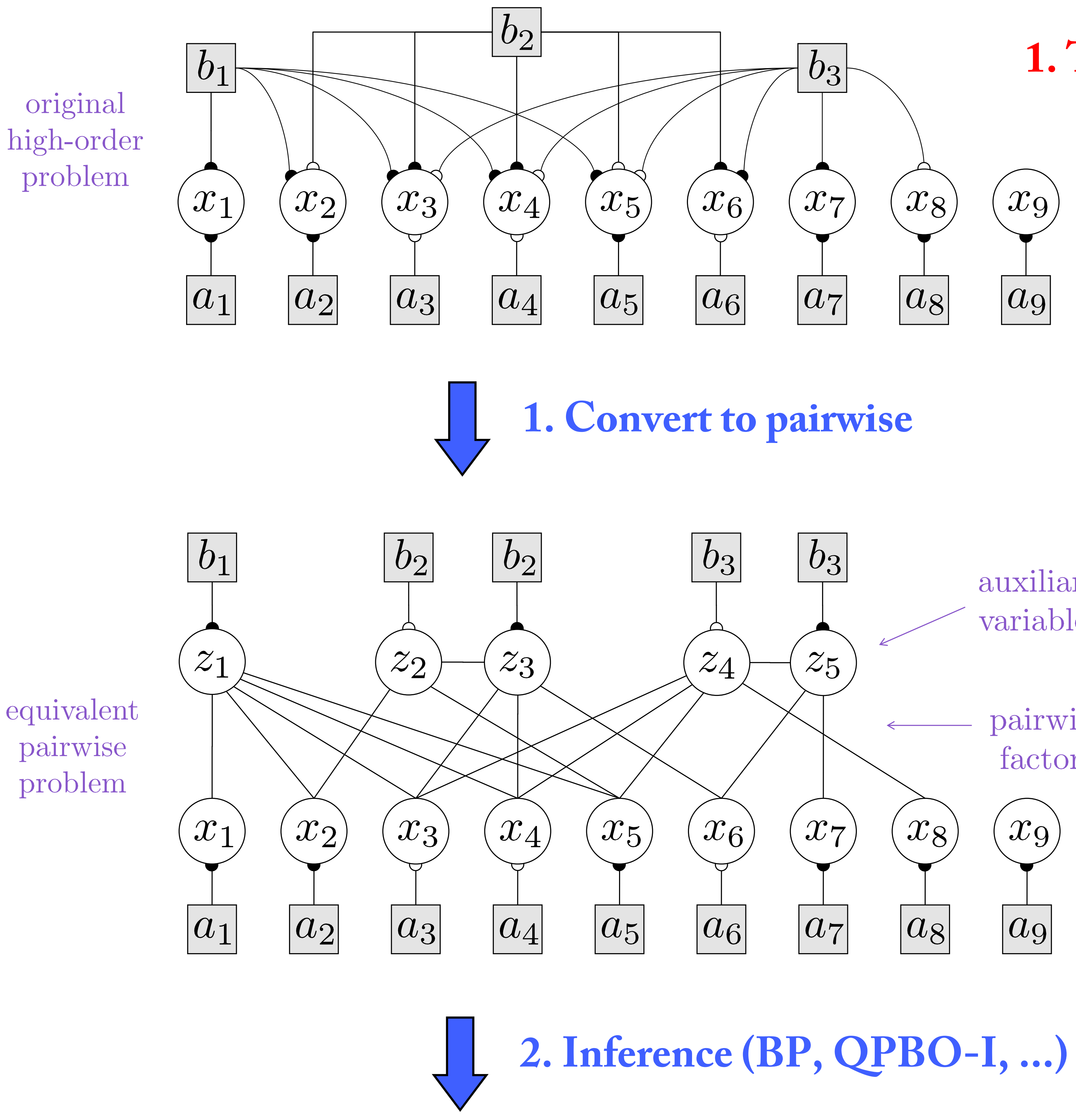
Submodular Vertex-Cover: Generalizes vertex-cover to allow *sub*modular costs [Iwata & Nagano, 2009].

$$\text{(SVC)} \quad \text{minimize } f(\boldsymbol{u}) \quad \text{subject to } u_i + u_j \geq 1 \quad \forall \{i,j\} \in \mathcal{E}, \quad u_j \in \{0,1\}.$$

submodular in $\boldsymbol{u}$

covering constraints

Typical Approach: Convert to *pairwise* problem with auxiliary variables [*e.g.* Rother *et al.*, 2009].



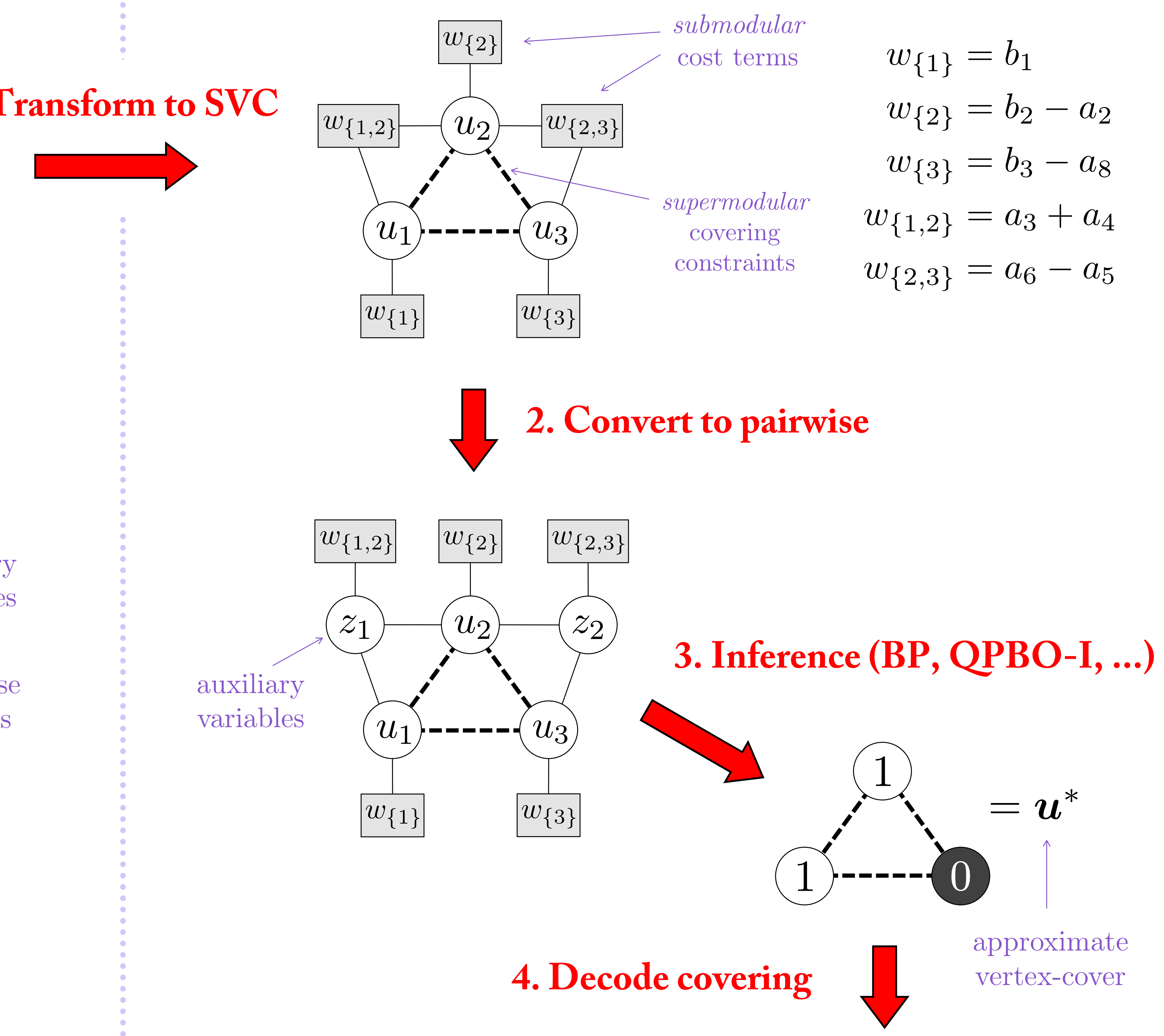
1. Convert to pairwise

2. Inference (BP, QPBO-I, ...)

$\boldsymbol{x}^* =$ 0 0 1 1 1 0 0 1 0

Our Approach: Transform to small SVC instance

1. Transform to SVC



$w_{\{1\}} = b_1$
$w_{\{2\}} = b_2 - a_2$
$w_{\{3\}} = b_3 - a_8$
$w_{\{1,2\}} = a_3 + a_4$
$w_{\{2,3\}} = a_6 - a_5$

2. Convert to pairwise

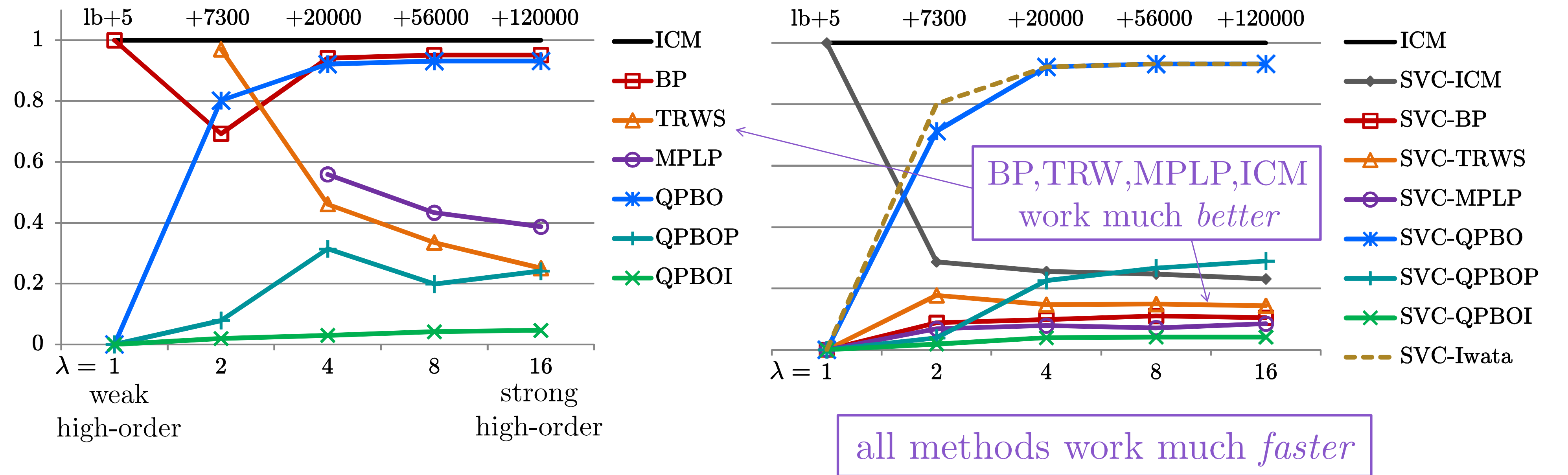
3. Inference (BP, QPBO-I, ...)

4. Decode covering

$\boldsymbol{x}^* =$ 0 0 1 1 1 0 0 1 0

Synthetic Tests: 100x100 grid, random unary potentials 50 random pattern potentials of strength λ.

Solution quality. Range [best lower bound, ICM energy] normalised to [0,1] for each λ.

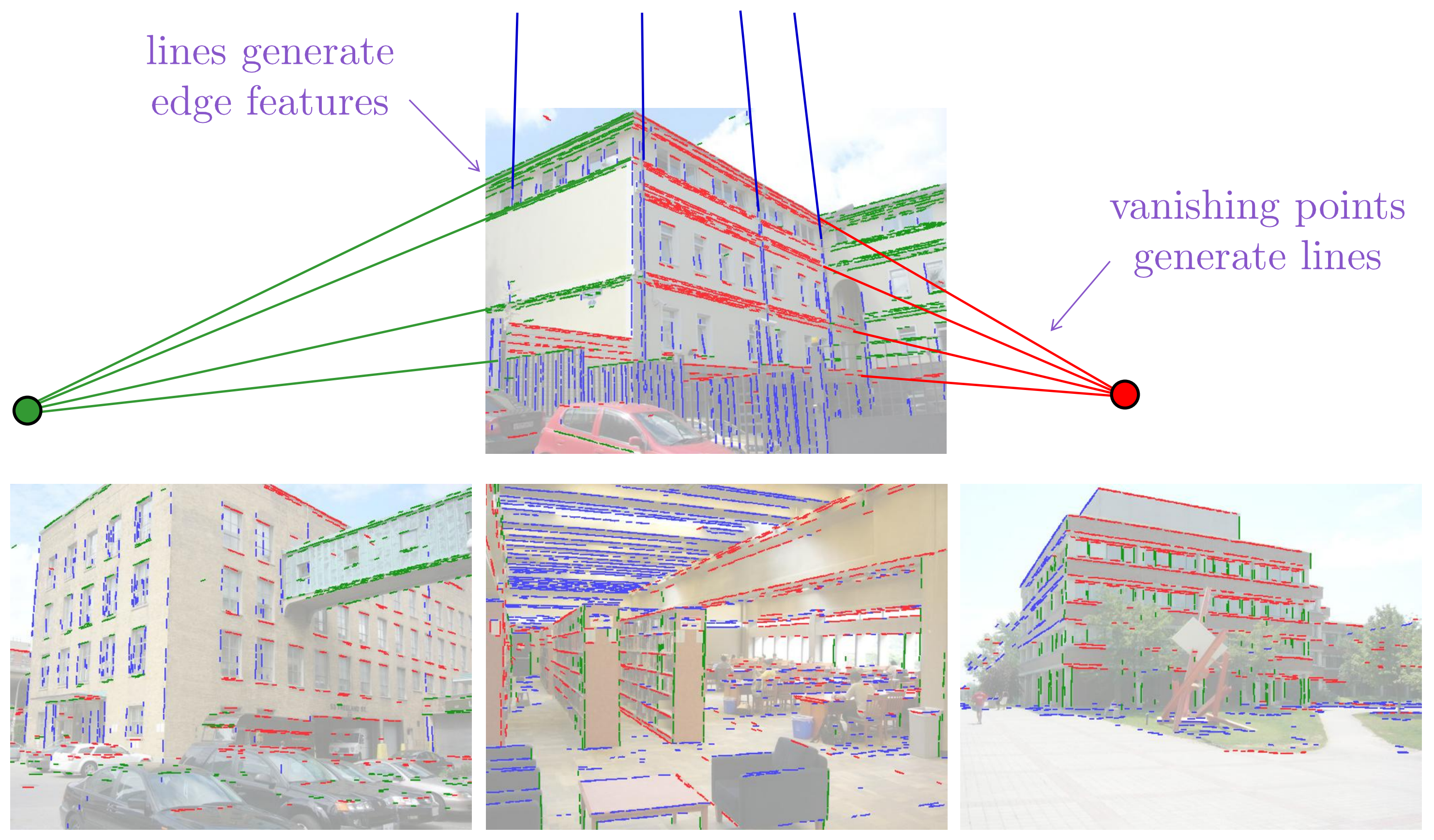


BP,TRW,MPLP,ICM work much *better*

all methods work much *faster*

Run times	BP	TRW-S	MPLP	QPBO	QPBO-P	QPBO-I	Iwata
direct	22ms	670ms	25min	30ms	25sec	140ms	N/A
as SVC	**5.2ms**	19ms	80sec	5.4ms	99ms	**7.2ms**	5ms

Application: Fast *fusion operation* for *hierarchical* model-fitting / clustering, *e.g.* scene parsing [Tretyak *et al.*, 2011].



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